

On the Mode Coupling in Longitudinally Magnetized Waveguiding Structures

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Abstract — The propagation in waveguiding structures containing gyro-magnetic material with longitudinal magnetization is analyzed in terms of the coupling between modes having different symmetry properties. It is found that the wave supported by a symmetrical structure of two coupled guides is a combination of the even and odd modes propagating in an isotropic structure. The gyromagnetic medium causes the coupling between the modes, and the energy of the wave is periodically transferred from one mode to another, which in turn results in the exchange of energy between guides. The mathematical model proposed here explains the operation of novel nonreciprocal devices which have been investigated experimentally by other researchers.

I. INTRODUCTION

RECENTLY, several papers have been published concerning nonreciprocal devices operating with a ferrite magnetized in the propagation direction [1]–[3]. The experimental results presented in [1] and [3] proved the feasibility of isolators and circulators based on certain phenomena occurring in the structures of two coupled guides with longitudinally magnetized gyromagnetic material. Since such waveguiding structures cannot violate the generalized reciprocity theorem [4], [5], investigators attributed the nonreciprocity observed in the experiment to stray transverse bias field components or to the inhomogeneity and asymmetry of the realized devices [5]. Recently, an attempt was made to create a mathematical model of the propagation in two dielectric image lines separated by a ferrite slab [2]. However, the results of this investigation cannot be accepted, since they were based on an erroneous interpretation of the solutions of the wave equation in a ferrite medium [9]. As a result, the model presented in [2] is incompatible with the theory of bidirectional gyromagnetic waveguides [8], and no satisfactory explanation of the effects causing the nonreciprocal behavior of the novel devices has been given yet.

In this paper, we propose a mathematical explanation of the effect observed in nonreciprocal devices whose construction is based on two coupled guides containing longitudinally magnetized ferrites. The analytical model of electromagnetic wave propagation is formulated using the coupled mode approach.

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II. ANALYSIS

Let us consider the electromagnetic field propagation in a lossless, uniform waveguiding structure filled with gyromagnetic medium whose physical parameters, characterized by a scalar relative permittivity ϵ and a tensor relative permeability $\vec{\mu}$, are functions of transverse coordinates. We assume that the propagation is in the z direction and that the electric and magnetic fields are proportional to the factor $e^{j\omega t}$, which will henceforth be suppressed for the sake of clarity. The direction of the dc magnetic field is taken to be parallel to the propagation direction.

The solution to the problem being analyzed can be found using the coupled mode method developed by Marcuse [6] and applied by Awai and Itoh [7] to the analysis of gyromagnetic structures with transverse magnetization. According to this procedure, the fields \vec{E} and \vec{H} in the investigated guide are expressed in terms of the fields of a second, basis waveguiding structure whose modal solutions \vec{E}_n , \vec{H}_n are known. This can be formally written as

$$\begin{aligned}\vec{E} &= \sum_n a_n(z) \vec{E}_n(x, y), \\ \vec{H} &= \sum_n a_n(z) \vec{H}_n(x, y).\end{aligned}\quad (1)$$

Assuming that the basis guide is filled with isotropic gyromagnetic material whose parameters are depicted by scalars ϵ and μ , and that the guide is otherwise identical to the investigated structure and following the procedure described in [7], we obtain the coupled mode equations:

$$\sum_n \left(P_{in} \frac{\partial}{\partial z} + jR_{in} \right) a_n = 0 \quad (2)$$

with

$$P_{in} = \int_{\Omega} \vec{a}_z \cdot (\vec{E}_n \times \vec{H}_i^* + \vec{E}_i^* \times \vec{H}_n) d\Omega \quad (3a)$$

$$R_{in} = \beta_n P_{in} + k_0 \eta_0 \int_{\Omega_0} \vec{H}_i^* (\vec{\mu} - \mu) \vec{H}_n d\Omega_0 \quad (3b)$$

where Ω and Ω_0 are the cross section of the analyzed guide and the cross section of the ferrite material, respectively, η_0 and k_0 are the intrinsic impedance and wavenumber in the free space, and β_n denotes the propagation constant of the n th mode in the basis structure.

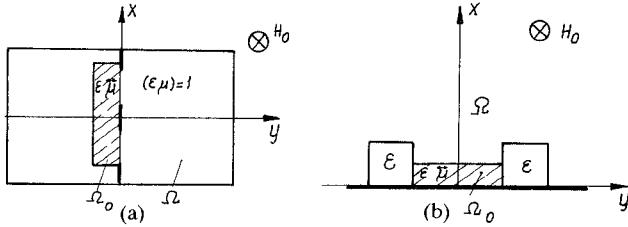


Fig. 1. Symmetrical structures containing longitudinally magnetized ferrite slabs. (a) Coupled-slot finline. (b) Coupled dielectric image lines.

The coefficients P_{in} vanish for $i \neq n$ owing to the orthogonality of the modal solutions in the basis structure, whereas the P_n 's equal four times the Poynting energy of the i th mode in the basis guide. Let us examine the coefficients R_{in} . For the ferrite magnetized in the z direction we obtain

$$\begin{aligned} \vec{\mu} - \mu &= \begin{bmatrix} \mu & j\mu_a & 0 \\ -j\mu_a & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{bmatrix} \\ &= \begin{bmatrix} 0 & j\mu_a & 0 \\ -j\mu_a & 0 & 0 \\ 0 & 0 & 1 - \mu \end{bmatrix}. \end{aligned} \quad (4)$$

Substituting (4) into (3b) we get

$$R_{in} = \beta_n P_{in} + C_{zin} + jC_{tin} \quad (5)$$

where

$$C_{zin} = k_0 \eta_0 (1 - \mu) \int_{\Omega_0} H_{zi}^* H_{zn} d\Omega_0 \quad (6a)$$

$$C_{tin} = k_0 \eta_0 \mu_a \int_{\Omega_0} (H_{xi}^* H_{yn} - H_{yi}^* H_{xn}) d\Omega_0. \quad (6b)$$

Let us discuss the quantities C_{tin} , representing the coupling between the modes in the basis guide due to the anisotropy of the ferrite. C_{tin} always vanishes if $n = i$. For $n \neq i$, C_{tin} becomes zero only if the structure is symmetric and the fields corresponding to the n th and i th modes are both even or odd. Conversely, for the symmetrical structure, if the i th and n th modes have different symmetry properties, $C_{tin} \neq 0$. For the analyzed guides this means that owing to the anisotropy properties of the longitudinally magnetized ferrite the even or odd excitation of the symmetric structures automatically initiates the propagation of odd or even modes, respectively. Hence, the effects observed in experimental devices should be studied in terms of phenomena caused by the coupling between modes supported by the structure.

III. COUPLING IN SYMMETRICAL STRUCTURES

We shall now apply the results obtained in the previous section to the analysis of symmetrical structures. We shall concentrate on the investigation of ferrite-loaded coupled-slot finline (Fig. 1(a)) and two dielectric image lines separated by a slab of a gyromagnetic medium (Fig. 1(b)). These structures have been recently used [1], [3] for the construction of nonreciprocal devices. The guides are sym-

metric with respect to either the plane $x = 0$ or the plane $y = 0$. We assume that basis structures support two orthogonal modes (even and odd) and only these modes are used in the field expansion to approximate fields in the investigated guides. Additionally, we assume that the ferrite is weakly magnetized and $\mu \approx 1$. Under these assumptions, the coupled mode equations (2) take the following form:

$$\begin{aligned} \left(P^{ee} \frac{\partial}{\partial z} + j\beta^e P^{ee} \right) a_e - C_i^{eo} a_o &= 0 \\ -C_i^{oe} a_e + \left(P^{oo} \frac{\partial}{\partial z} + j\beta^o P^{oo} \right) a_o &= 0 \end{aligned} \quad (7)$$

where $C_i^{oe} = -C_i^{eo*}$ and β^e, β^o denote the propagation constants of the even and the odd mode in the basis structure, respectively. Normalizing the fields in the basis line according to the condition

$$\int_{\Omega} (\vec{E}_i \times \vec{H}_i^*) d\Omega = 1$$

we obtain

$$\begin{aligned} \frac{\partial}{\partial z} a_e &= -j\beta^e a_e + C a_o \\ \frac{\partial}{\partial z} a_o &= -j\beta^o a_o - C^* a_e \end{aligned} \quad (8)$$

where

$$C = \frac{1}{2} k_0 \eta_0 \mu_a \int_{\Omega_0} (H_x^* H_y^o - H_y^* H_x^o) d\Omega_0 \quad (9)$$

Taking the propagation in the analyzed guide with the factor e^{-jkz} , we rewrite (8) in the following form:

$$\begin{aligned} (k - \beta^e) a_e - jCa_o &= 0 \\ jCa_e + (k - \beta^o) a_o &= 0. \end{aligned} \quad (10)$$

The propagation constant k in the investigated guide is found by setting to zero the determinant of the above set of equations. Solving the resulting quadratic equation yields

$$k_{1/2} = \frac{\beta^e + \beta^o}{2} \pm \sqrt{\left(\frac{\beta^e - \beta^o}{2} \right)^2 + |C|^2} = \beta_0 \pm \Gamma \quad (11)$$

where

$$\beta_0 = \frac{\beta^e + \beta^o}{2} \quad \Gamma = \sqrt{\Delta\beta^2 + |C|^2} \quad \Delta\beta = \frac{\beta^e - \beta^o}{2}.$$

Let us now examine, for different line excitations, the propagation of the two coupled modes in the positive z direction. First of all we define the even mode in the basis structure as having the electric field E_x an even function with respect to the symmetry plane, and the odd mode having E_x an odd function with respect to the symmetry plane. The orientation of the electric field vectors for both modes in image and finline configurations is shown schematically in Fig. 2. Note that the converse definition of modes was adopted in [1].

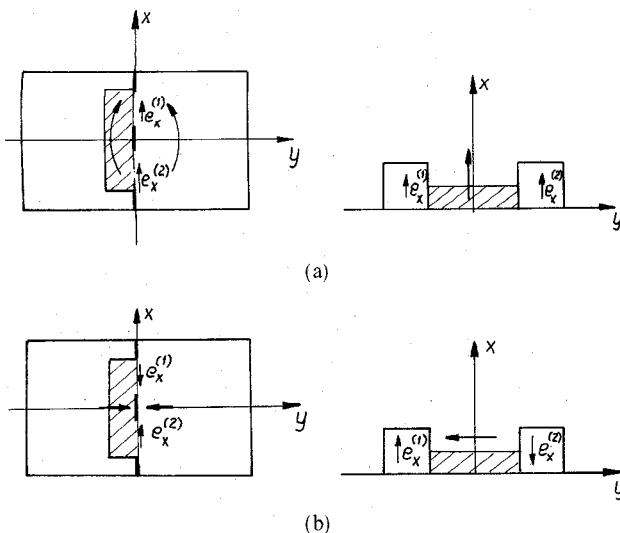


Fig. 2. Orientation of electric field for (a) even-mode excitation and (b) odd-mode excitation.

The general solutions to differential equations (8) can be written as

$$\begin{aligned} a_e(z) &= A_1 e^{-jk_1 z} + A_2 e^{-jk_2 z} \\ a_o(z) &= -j(m_1 A_1 e^{-jk_1 z} + m_2 A_2 e^{-jk_2 z}) \end{aligned} \quad (12)$$

with

$$m_1 = \frac{k_1 - \beta^e}{C} = \frac{\Gamma - \Delta\beta}{C} \quad m_2 = \frac{k_2 - \beta^e}{C} = -\frac{\Gamma + \Delta\beta}{C}.$$

We assume first that the structure was excited in the plane $z = 0$ by the even mode; i.e., $a_e(0) = 1$, $a_o(0) = 0$. For this excitation we obtain

$$\begin{aligned} a_e(0) &= A_1 + A_2 = 1 \\ a_o(0) &= m_1 A_1 + m_2 A_2 = 0. \end{aligned} \quad (13)$$

Solving (13) for the unknown amplitudes A_1 and A_2 , we obtain

$$\begin{aligned} A_1 &= \frac{1}{2} \left(1 + \frac{\Delta\beta}{\Gamma} \right) \\ A_2 &= \frac{1}{2} \left(1 - \frac{\Delta\beta}{\Gamma} \right). \end{aligned} \quad (14)$$

Substituting the above expressions into (12), we obtain

$$\begin{aligned} a_e(z) &= \frac{1}{2} \left[\frac{\Gamma + \Delta\beta}{\Gamma} e^{-j(\beta_0 + \Gamma)z} + \frac{\Gamma - \Delta\beta}{\Gamma} e^{-j(\beta_0 - \Gamma)z} \right] \\ a_o(z) &= \frac{-j(\Gamma^2 - \Delta\beta^2)}{2C\Gamma} [e^{-j(\beta_0 + \Gamma)z} - e^{-j(\beta_0 - \Gamma)z}]. \end{aligned} \quad (15)$$

The above equations can be transformed to the more convenient form

$$\begin{aligned} a_e(z) &= \left[\cos(\Gamma z) - j \frac{\Delta\beta}{\Gamma} \sin(\Gamma z) \right] e^{-j\beta_0 z} \\ a_o(z) &= -\frac{C}{\Gamma} \sin(\Gamma z) e^{-j\beta_0 z} \end{aligned} \quad (16)$$

We shall now examine the behavior of the e_x field component in each guide (slot or image line) constituting the investigated structure. According to the definition adopted in this paper, the fields in both guides for the even mode in the basis structure are equal and in phase ($e_x^{(1)} = e_x^{(2)} = E$, superscripts 1 and 2 denoting the guide), and for the odd mode they have equal amplitudes but are 180° out of phase ($e_x^{(1)} = -e_x^{(2)} = E$). Therefore the field in guide 1 can be written as

$$e_x^{(1)}(z) = E [a_e(z) + a_o(z)]$$

and that for guide 2 can be written as

$$e_x^{(2)}(z) = E [a_e(z) - a_o(z)]$$

or, using (16),

$$\begin{aligned} e_x^{(1)}(z) &= E \left[\cos(\Gamma z) \mp \frac{|C|}{\Gamma} \sin(\Gamma z) - j \frac{\Delta\beta}{\Gamma} \sin(\Gamma z) \right] e^{-j\beta_0 z} \\ e_x^{(2)}(z) &= E \left[\cos(\Gamma z) \pm \frac{|C|}{\Gamma} \sin(\Gamma z) - j \frac{\Delta\beta}{\Gamma} \sin(\Gamma z) \right] e^{-j\beta_0 z}. \end{aligned} \quad (17)$$

The choice of sign in the above formulas depends on the magnetization direction. The expressions for the odd excitation can be derived in the same way.

$$\begin{aligned} e_x^{(1)}(z) &= E \left[\cos(\Gamma z) \pm \frac{|C|}{\Gamma} \sin(\Gamma z) - j \frac{\Delta\beta}{\Gamma} \sin(\Gamma z) \right] e^{-j\beta_0 z} \\ e_x^{(2)}(z) &= E \left[-\cos(\Gamma z) \pm \frac{|C|}{\Gamma} \sin(\Gamma z) \right. \\ &\quad \left. + j \frac{\Delta\beta}{\Gamma} \sin(\Gamma z) \right] e^{-j\beta_0 z}. \end{aligned} \quad (18)$$

If the structure is fed only from one guide, the field in each guide can be obtained as a superposition of fields derived for even and odd excitations. For instance, the excitation in guide 1 yields

$$\begin{aligned} e_x^{(1)} &= E \cos(\Gamma z) \\ e_x^{(2)} &= E \frac{\pm |C| - j\Delta\beta}{\Gamma} \sin(\Gamma z). \end{aligned} \quad (19)$$

From the above equation it is seen that as the wave propagates, its energy is periodically exchanged between the guides. The strongest coupling effect occurs for $\beta^e = \beta^o$. In this case, after normalization ($E = 1$) we obtain

for the even excitation:

$$\begin{aligned} e_x^{(1)} &= \cos(Cz) \mp \sin(|C|z) \\ e_x^{(2)} &= \cos(Cz) \pm \sin(|C|z) \end{aligned} \quad (20)$$

for the odd excitation:

$$\begin{aligned} e_x^{(1)} &= \cos(Cz) \pm \sin(|C|z) \\ e_x^{(2)} &= -\cos(Cz) \pm \sin(|C|z) \end{aligned} \quad (21)$$

for the excitation in guide 1:

$$\begin{aligned} e_x^{(1)} &= \cos(Cz) \\ e_x^{(2)} &= \pm \sin(|C|z). \end{aligned} \quad (22)$$

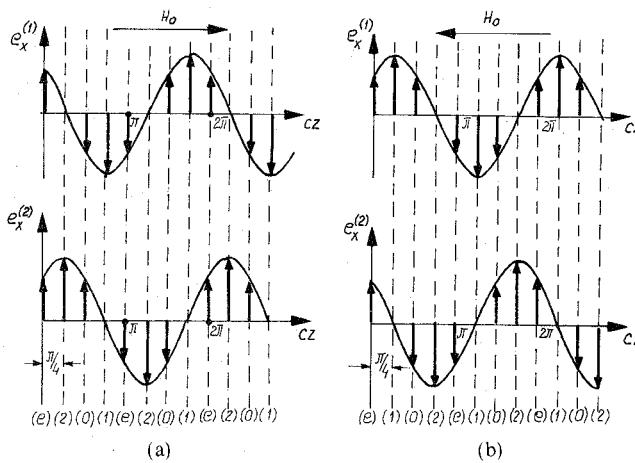


Fig. 3. Amplitude of electric field in both guides for $\beta^e = \beta^o$. (a) Magnetization in positive z direction. (b) Reverse magnetization. (Symbols e , o , 1, and 2 denote the planes of excitation by the even and odd modes or from guide 1 and guide 2, respectively.)

Fig. 3 shows the plots of the field amplitudes in each guide for two opposite directions of the static magnetic field. From the diagram we may observe that:

- As the wave propagates, the energy of one mode goes to the other. Over the distance $Cz = \pi/2$ we observe the total exchange of energy between the modes.
- If the structure is excited by the even mode, the field in guide 1 vanishes at $Cz = \pi/4$ from the excitation plane and the field is concentrated in guide 2. The converse effect occurs if the biasing magnetic field is reversed.
- The odd excitation causes an effect similar to the change of magnetization direction in case b). Over the distance $Cz = \pi/4$ from the excitation plane the field in guide 2 becomes zero and is maximal in guide 1. Again the change of magnetization direction results in the converse effect.

IV. FARADAY ROTATION MODEL

We will now introduce an alternative model of the phenomena discussed in the previous section. For this purpose we have to recall the definition of the modes in the basis structure adopted in this paper. According to Fig. 2, the electric field in the symmetry plane is either in phase (even mode) or 90° out of phase (odd mode) with respect to the e_x field components in each guide. We will henceforth refer to this field as the excitation vector. Let us consider the excitation vector in the two planes 1-1' and 2-2' shown in Fig. 4. We have observed in the previous section (cf. Fig. 3) that beyond the distance $Cz = \pi/2$ the even mode becomes the odd mode. This means that as the wave propagates from plane 1-1' to 2-2' (Fig. 4(a)), the excitation vector is rotated by 90° in the clockwise direction. If the propagation direction is reversed (Fig. 4(b)), the excitation vector continues to rotate in the same direction and its original polarization direction is not restored. This effect is identical to the Faraday rotation phe-

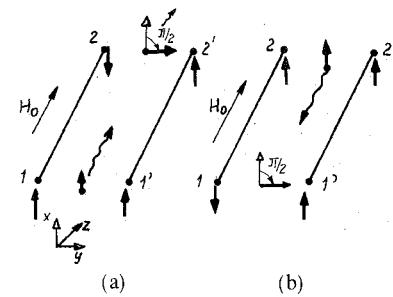


Fig. 4. Rotation of the excitation vector over the distance $Cz = \pi/2$. (a) Forward propagation. (b) Backward propagation.

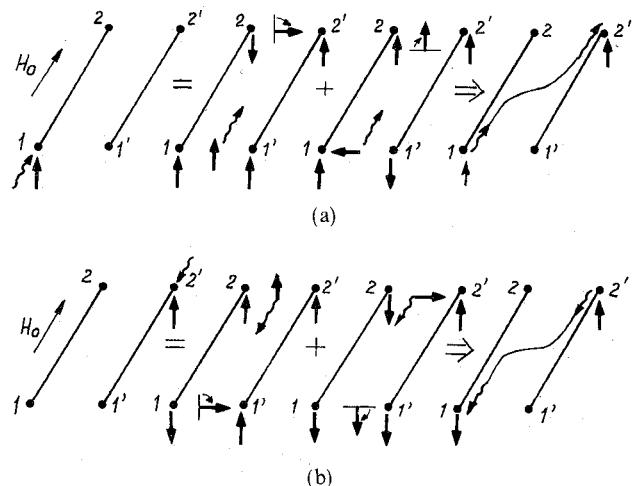


Fig. 5. Propagation in coupled lines of length $Cz = \pi/2$. (a) Excitation in port 1. (b) Excitation in port 2'.

nomenon observed for the linearly polarized TEM wave in an unbounded gyromagnetic medium magnetized in the propagation direction.

Using the new model, which is more suitable for the analysis of phase relations between field vectors, let us now investigate the propagation of the wave incident in port 1. Fig. 5(a) shows that the partial waves from even and odd excitation will combine and emerge from port 2'. When the propagation is from 2' to 1 (Fig. 5(b)), the wave will arrive at port 1 with its phase changed by 180° . Hence, two coupled lines of length $Cz = \pi/2$ with matched ports 1' and 2 become a gyrator.

If the distance between planes 1-1' and 2-2' is reduced to $Cz = \pi/4$, the excitation vector undergoes a rotation by $\pi/4$. Its E_x and E_y components initiate the propagation of even and odd modes, respectively. Thus, the excitation in port 1 (Fig. 6(a)) will result in the even mode emerging at plane 2-2'. The wave incident in port 2' (Fig. 6(b)) will produce at the plane 1-1' the field distribution corresponding to the odd mode.

The above observations entirely agree with the results of coupled mode analysis summarized in Fig. 3. Since the Faraday rotation model is clearer, it will serve to explain the operation of nonreciprocal devices using coupled guides with longitudinally magnetized ferrites.

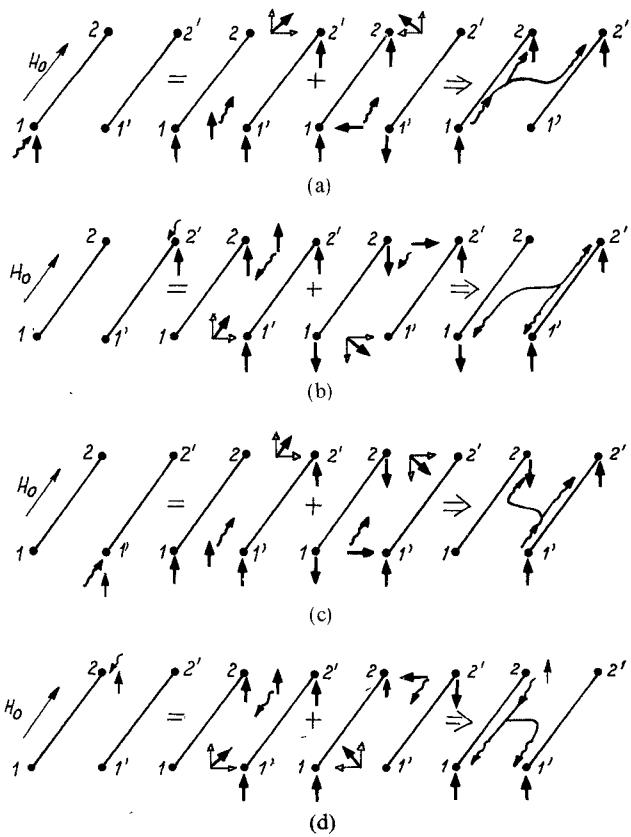


Fig. 6. Propagation in coupled lines of length $Cz = \pi/4$. Excitation in port: (a) 1, (b) 2', (c) 1', and (d) 2.

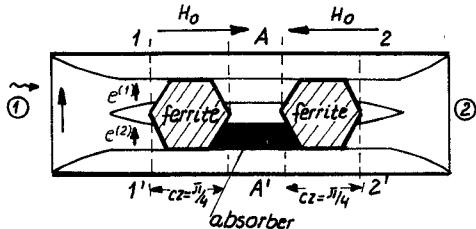


Fig. 7. The geometry of a coupled-slot finline isolator with two ferrite slabs magnetized in opposite directions.

V. OPERATION OF COUPLED MODE NONRECIPROCAL DEVICES

Having discussed the phenomenon of coupling between modes with different symmetry properties, we shall now explain the nonreciprocal behavior of the experimental devices investigated by other researchers [1]. Fig. 7 shows the structure of the isolator in finline technology. Note that a resistive card is placed over one slot between two ferrites magnetized with antiparallel magnetic fields. The transition from a unilateral single-slot finline into a coupled line region ensures the excitation of the even mode (the odd mode according to the convention used in [1]). Fig. 8 explains the operation of the isolator. Using the Faraday rotation model discussed earlier, we may conclude that the even excitation in the plane 1-1' will result in transmission of the wave to port A', where it will be absorbed in the resistive card. For the reverse propagation,

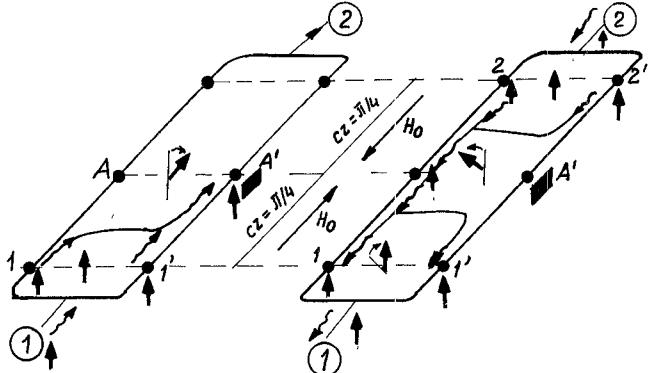


Fig. 8. Operation of the isolator depicted in Fig. 7.

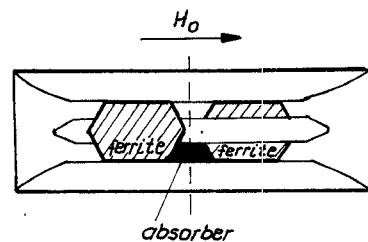


Fig. 9. Isolator with ferrites placed on opposite sides of the conductor.

the excitation vector corresponding to the even mode will be rotated counterclockwise by 45° and the wave will be transmitted to port A, thus bypassing the resistive card placed over the other slot. Since the two sections are magnetized with antiparallel magnetic fields, the vertical polarization of the excitation vector will be restored and the fields in ports 1 and 1' will have the right orientation to combine and emerge from the output. The direction of the rotation depends on the sign of coupling coefficient C defined by (9). Therefore, the same reasoning explains the operation of the isolator with the alternative arrangement of ferrites also investigated in [1]. In this arrangement one ferrite is moved to the other side of the conductor (Fig. 9) and a single unidirectional field is applied. For both the even and the odd mode, the x magnetic field components on both sides of the conductor are 180° out of phase. Since the formula defining the coupling coefficient C involves integrating over the cross section of the ferrite expressions whose sign depends on the sign of H_x^o and H_x^e , the change in location of the ferrite reverses the sign of C . This effect is entirely equivalent to the change of magnetization direction.

The discussion presented above proves that the theory proposed in this paper allows us to explain the results of measurements of experimental isolators realized in finline technology. However, we have not succeeded in explaining the operation of a four-port circulator which would be based on a simple section of two parallel uniform coupled guides. The only effect we predicted was the nonreciprocal phase change discussed in Section IV. Nonetheless, circulation can be obtained in the structures shown in Fig. 10. The operation of a three-port circulator (Fig. 10(a)) is as follows: A wave incident at port 1 excites ports 2' and 2''

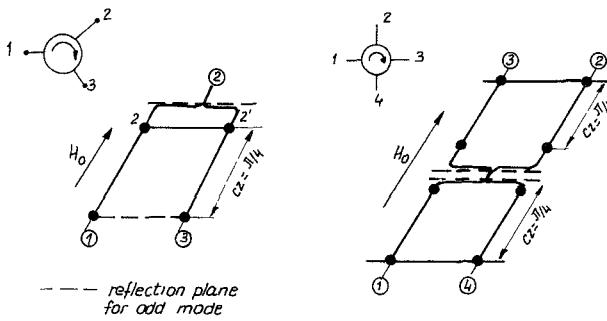


Fig. 10. Three- and four-port circulators. (a) Operation of a three-port circulator. (b) Two cascaded three-port circulators operating as a four-port circulator.

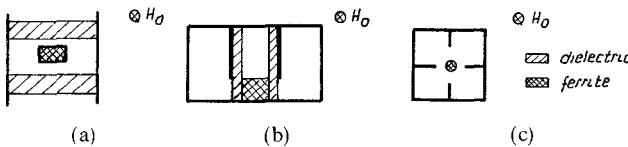


Fig. 11. Exemplary waveguiding structures for which the mode coupling phenomenon may be used to construct novel nonreciprocal devices operating with ferrite magnetized in the propagation direction. (a) H guide. (b) Bilateral finline. (c) Crossed finlines in a waveguide of square cross section.

in phase (cf. Fig. 6(a)). The fields from these two arms will combine and emerge from port 2. A wave incident at port 2 will excite the even mode in plane $2'-2''$ and the wave will be transmitted to port 3 (cf. Fig. 6(b)). For the excitation in port 3, the field vectors in ports $2'$ and $2''$ will be 180° apart in phase (cf. Fig. 6(c)) and, in consequence, the wave will be reflected from port 2. The reflected wave will excite the odd mode in plane $2'-2''$, and accordingly the wave will be transmitted to port 1 (cf. Fig. 6(b)). The change of magnetization direction changes the circulation direction. The four-port circulator illustrated in Fig. 10(b) consists of two cascaded three-port circulators.

The mathematical model of effects observed in symmetrical structures of two coupled transmission lines containing a longitudinally magnetized ferrite was derived under a few simplifying assumptions. For instance, we have neglected the coupling with waves traveling in the reverse direction. The discussion of the operation of nonreciprocal devices was based on the Faraday rotation model, which was derived under the assumption that the odd and even modes have equal phase velocities. The difference in propagation constants will prevent the total exchange of energy between modes and guides. A full analysis of the problem, taking into account the aforementioned factors, would give a better picture of the mode coupling phenomenon and perhaps reveal interesting new effects which could be employed in the construction of nonreciprocal devices.

The results of the analysis indicate that the phenomena discussed in this paper will occur in any waveguiding structure loaded with gyromagnetic material magnetized in the propagation direction, provided that two modes of different symmetry properties can be excited. Fig. 11 shows

a few structures of interest from the point of view of possible applications. Work is under way to compute the coupling coefficient C for some of the structures shown in Fig. 11 and to create a design technique which will make it possible to predict the transmission characteristics of novel nonreciprocal devices.

VI. CONCLUSIONS

A mathematical model of propagation in structures supporting two modes with different symmetry properties and containing longitudinally magnetized gyromagnetic material is presented. It was found that:

- The wave propagating in such structures consists of even and odd modes existing in isotropic lines.
- The even and odd modes are coupled.
- The coupling between basis modes is due only to the anisotropy of the gyromagnetic medium.
- As the wave propagates, the energy is periodically transferred from one basis mode to another.
- The mode coupling results in the exchange of energy between guides.
- If basis modes are degenerate, the coupling between modes can be analyzed in terms of a Faraday rotation phenomenon.

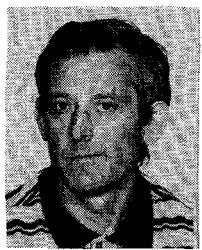
The theory proposed in this paper gives an excellent explanation of the operation of experimental nonreciprocal devices and is compatible with the theory of bidirectional gyrotropic waveguides [8].

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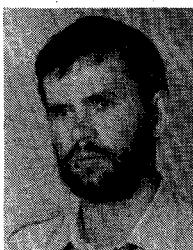
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wave applications.

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